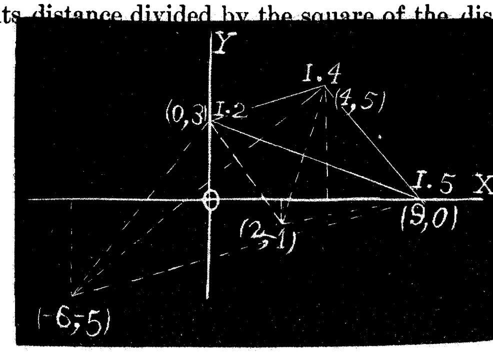
Taking the equation formed by the 1st and 2nd $x^2+8x+y^2-2y-23=0$, is the locus of points equally illuminated by the lights of 2 and 4 intensities. The 1st and 3rd give $x^2+12x+y^2-10y-39=0$ as being points equalluminated by lights 2 and 5. The 2nd and 3rd give $x^2+32x+y^2-50y-119=0$ for lights 4 and 5. The points equally illuminated by all are found by making these simultaneous as $(2,-1)$ and $(-6,-5)$.

Let x, y be the co-ordinates of the point sought.

Then since intensity at units distance divided by the square of the distance to any point is the intensity at that point, we get



$$\frac{2}{(3-y)^2+x^2}=\frac{4}{(4-x)^2+(5-y^2}$$

$$=\frac{5}{(9-x)^2+y^2}$$

$\therefore\ 39=x^2+y^2+12x-10y$ or

$x=-6\pm\sqrt{75-y^2+10y}$;

$23=x^2+y^2+8x-2y$ or

$x=-4\pm\sqrt{39-y^2+2y}$.

$\therefore\ -6\pm\sqrt{75-y^2+10y}=$

$-4\pm\sqrt{39-y^2+2y}$ or $y^2+6y=-5$.

$\therefore\ y=-1$ or -5, $x=2$ or -6.

$\therefore$ There are two points $(2,-1)$, $(-6,-5)$ both on the line $x-2y=4$.

Solutions to this problem were also received from *J. R. BALDWIN, P. H. PHILBRICK, LEONARD E. DICKSON*, and *T. W. ATKINSON*. An excellent solution accompanied by a beautiful blue-print figure was also received, but the author failed to sign his name to his work.

17. Proposed by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana University, Bloomington, Indiana.

Draw a circle bisecting the circumference of three given circles.

Solution by HENRY HEATON, M. S., Atlantic, Iowa.

Let A and B be the centers of any two circles. Join AB and draw diameters DC and EF perpendicular thereto. Through P, the center of the circle which would pass through D, C, E, and F, draw QR, perpendicular to AB.

From L, any point of QR, draw LA, LB, and the diameters GH and KI respectively perpendicular to LA and LB. Then will $LC=LH=LI=LK$.

For $PD=PE$ by construction. $\therefore\ PA^2+AD^2=PB^2+BE^2$.

Substituting AC^2 for its equal AD^2, BK^2 for its equal BE^2, and adding PL^2 to both members of the equation, we get $PL^2+PA^2+AC^2=PL^2+PB^2+BK^2$. But the first member of this equation equals LC^2 and the second member equals LK^2.

$\therefore\ LC=LK$. But $LC=LH$ and $LK=LI$.

$\therefore\ LC=LH=LI=LK$. Since GH and IK are diameters, if a circle be passed through G, H, I, and K it will bisect the circumferences of the circles A and B.

Since L is any point of QR, QR is the locus of centers of circles which will bisect the circumferences of the circles A and B.

The required construction is now obvious. Draw the locus of centers of circles which bisect the circumferences of the given circles, taken two and two. Their common intersection will be the center of the required circle.

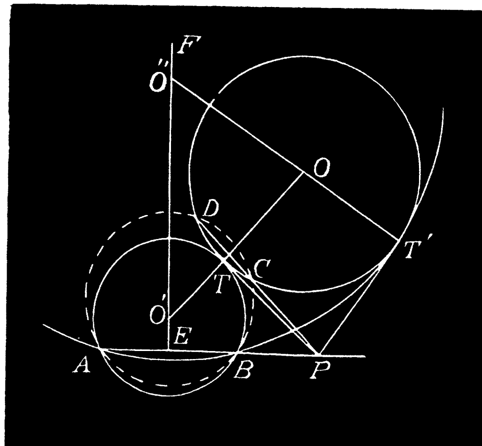
Solutions to this problem were also received from *Professors H. W. DRAUGHON, WILLIAM HOOVER, WILLIAM SYMMONDS, P. H. PHILBRICK* and ——

18. Proposed by Professor HENRY HEATON, Atlantic, Iowa.

Through two given points to draw two circles tangent to a given circle.

Solution by Professor T. A. TIMMONS, St. Mary's Kentucky; D. A. ROTHRICK. A. M., Professor of Mathematics, Indiana University, Bloomington, Indiana; J. H. BEACH, Tiffin, Ohio; P. S. BERG, Apple Creek, Ohio; JOHN DOLLMAN, Jr., Councellor at Law, Philadelphia, Pennsylvania; and P. H. PHILBRICK, C. E., Lake Charles, Louisana

Let A and B be the given points and O the center of the given circle. Through the points A and B draw *any* circle cutting the given circle in the points C and D. Draw the lines AB and DC and produce them until they meet in P. Draw the tangents PT and PT'. At E, the middle point of AB, erect the perpendicular EF. Now EF is the locus of the center of all circles passing through A and B. Draw the radius OT and produce it till it meets EF in O'. Then O' is the center of a circle passing through A and B and tangent to the given circle at T.



In like manner, by drawing the radius OT' and producing it to meet EF, we find O'', the center of another circle fulfilling the conditions of the problem.

Discussion.—If one of the given points is within the given circle, the point P falls within the given circle and there is no solution. If the given points lie on the circumference of the given circle there is one solution. In all other cases there are two solutions. When EF passes, through the center of the given circle the general construction fails.

This problem was solved in a similar manner by *G. B. M. ZERR, WILLIAM SYMMONDS*, and *The PROPOSER*. *H. C. WHITAKER* solved it by Cartesian Geometry. *CHRISTIAN HOMING, Professor of Mathematics, in Herdleberg College*, did not solve the problem but referred to Casey's Sequel to Euclid's Elements, Prop. X., Sect. V., Bk. VI.